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Analysis of Distributed Resources Operating in Unbalanced Distribution Circuits

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Abstract- The use of Distributed Resource (DR) electrical generation in distribution circuits is growing. DR generation comes in many forms including gas turbine driven synchronous generators, wind powered induction generators, fuel cells with inverter circuitry, and others. This work focuses on the operation of DRs in unbalanced distribution circuits. It is shown that a balanced DR, modeled as a voltage source, can cause unbalanced power flows in a distribution circuit with balanced loads. A three-phase model of a synchronous generator operating as a DR is derived. This derivation is used to justify a voltage-dependent, current source model of the DR which is then used in subsequent power flow studies. A power flow study is presented where a circuit is operating with no constraint violations. A DR is then switched into the circuit, and following this switching operation, voltage constraint violations occur on some phases.

Keywords: Distributed resources, generation and storage, distribution planning, unbalanced power flow.

I. INTRODUCTION

The use of Distributed Resource (DR) electrical generation in distribution circuits is growing. DR generation comes in many forms including gas turbine driven synchronous generators, wind powered induction generators, fuel cells with inverter circuitry, and others. The economic and technical advantages of DRs are being considered in residential, commercial, and industrial market places [1, 2].

During normal conditions distribution circuits operate in unbalanced states, with at a given location secondary voltage magnitudes deviating between phases by as much as 8 volts or more on a 120 volt basis. This paper investigates power flow analysis of DRs in association with distribution circuit imbalances, where imbalances arise from multi-phase circuit construction, unbalanced loads, and non-symmetric circuit impedances and admittances. For accurate analysis of distribution systems, the imbalances must be taken into account. This implies the use of a multiphase model for analysis that simulates each current path and each phase load.

Two circuit models of DRs are presented in this paper. The first model (section II) considers the DR operating as a three-phase voltage source. This is not the most accurate model of a DR, but it is considered in order to investigate and illustrate unbalanced effects. The second DR model (sections III-V) is derived just for synchronous generators, and it models the DR as a voltage-dependent current source.

When modeled as a three-phase, balanced voltage source, it is shown that the DR can cause unbalanced power flows in a distribution circuit with balanced loads. A circuit model is especially constructed to illustrate this point. The model contains two sources, the substation and the DR, and consists of eight miles of representative distribution construction. Both the substation and DR are balanced sources. Futhermore, all loads throughout the circuit are balanced. The circuit model is constructed such that when the voltages of both sources are the same, currents and voltages throughout the circuit are balanced.

Next the voltage of the DR source is caused to deviate from that of the substation. The deviation is performed such that the DR source remains balanced. When this deviation in source voltage occurs, voltages and currents throughout the circuit become unbalanced, and unbalanced power flows result. The unbalanced power flows are observed to be significant and are due to the unbalanced circuit impedance. The observations from this calculation are used to set up the interaction that occurs in a power flow solution with a more accurate DR model.

A three-phase model of a DR operating as a synchronous generator is developed. For simplicity, this model assumes a smooth rotor. The model is used to show that the currents injected by this particular type of DR may be expressed as a function of the difference between the terminal voltages and the field voltage of the machine. This model may then be used in a power flow solution.

For each iteration of the power flow solution, the currents injected by the DR may be calculated given the terminal voltages and the machine field voltage. Considered in this form, the DR is looked upon as a voltage-dependent, current source. The terminal voltages of the DR vary with each iteration of the power flow solution, and the currents injected by the DR depend upon the terminal voltages. The currents injected by the DR will affect the results from the next iteration of the power flow solution. This is continued until convergence occurs.

A power flow study is run on a simple, but unbalanced distribution circuit model in which it is easy to verify results by hand. Analytical expressions are also used to verify the observed results.

Several conclusions from the studies are drawn, and areas for further work are considered.

II. THREE-PHASE BALANCED VOLTAGE SOURCE MODEL

In electrical networks and power system analysis, common models for sources, such as synchronous generators, are voltage or power sources. The general literature uses single-phase equivalent models. Previous system studies involving DRs have made use of single-phase equivalent models. [1, 2] Due to unbalanced conditions in distribution systems, more accurate analysis may be achieved by using multi-phase models. Section III of this paper will derive a multi-phase model for a synchronous generator operating as a DR.

In this section a generic model of a DR as a 3-phase balanced voltage source is considered. The assumption for this model is that for small deviations the DR is strong enough to control the voltage to any value and supply the resulting power flows. Observations of unbalanced effects in distribution circuits will be made with this model.

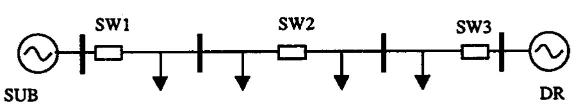

Figure 1. DR Modeled as Three-Phase Voltage Source in a Circuit with Balanced Loads and Unbalanced Impedances

In Figure 1 the substation has a single distribution feeder and the DR is placed at the very end of the feeder. The distribution feeder is modeled with three switches labeled as SW1 – SW3, and four line sections. Each line section is a multi-grounded, three-phase section with 336ACSR phase conductors and a 4/0 ACSR neutral return. A horizontal line spacing with A-B spacing of 0.76 meter (2.5 feet) and A-C spacing of 2.23meters (7.3 feet), with the neutral 0.91meter (3 feet) below the B phase, is used for the construction. The line model used here is representative of distribution systems. The length of each line section is 3048 meters (10000 feet), making the overall length of the distribution feeder approximately 12.228 km (7.6 miles). The phase impedance matrix, where impedances are specified in ohms, for each line section is given by

$$\left[z_{ij}\right] = \begin{bmatrix} 0.74+j1.98 & 0.27+j0.89 & 0.26+j0.67 \\ 0.27+j0.89 & 0.76+j1.90 & 0.27+j0.72 \\ 0.26+j0.67 & 0.27+j0.72 & 0.74+j1.98 \end{bmatrix} \quad (1)$$

Each line section supplies a three-phase load of 100 kW + j100 kVar or the overall feeder is supplying a total three-phase load of approximately 1.7 MVA. The only unbalance in the distribution circuit is due to the line impedance.

First assume that the voltage magnitude and angle of the DR are equal, for each corresponding phase, to those of the substation. In this case balanced voltages and currents exist throughout the feeder, with the substation supplying the loads on line sections between SW1 and SW2 and the DR supplying the loads on line sections between SW2 and SW3.

Next assume that the voltage magnitude of each phase of the DR deviates from the substation nominal by the same amount. In this case the DR is still operating as a balanced source, but its values are slightly different from those of the substation. Also in this case unbalanced voltages, currents, and power flows exist throughout the feeder.

To calculate the resulting voltages and currents, a distribution power flow program was used [3,4]. Tables 1 and 2 present results of power flow runs and show power flows at the DR as a function of deviations in DR voltage magnitudes and angles, where for magnitude deviations a 1% difference indicates that the DRs voltage magnitude is running 1% above that of the substation.

Table 1 DR Real Power Flows as a Percentage of the Total Load for Phases A, B, and C and Real Power Flow Imbalance Versus Percentage Change in DR Source Voltage Magnitude for Circuit of Figure 1

ΔV (% of nominal)	Pa (% of total load)	Pb (% of total load)	Pc (% of total load)	Average (% of total load)	Imbalance (%)
0.0	49.8	49.9	49.8	49.8	0.1
1.0	63.4	60.8	57.9	60.7	4.6
2.0	77.3	71.9	66.2	71.8	7.8
3.0	91.5	83.2	74.5	83.1	10.3
4.0	105.9	94.7	83.1	94.6	12.1
5.0	120.6	106.5	91.8	106.3	13.6

Table 2 DR Real Power Flows as a Percentage of the Total Load for Phases A, B, and C and Real Power Flow Imbalance Versus Change in DR Source Voltage Angle for Circuit of Figure 1

Δθ (degrees)	Pa (% of total load)	Pb (% of total load)	Pc (% of total load)	Average (% of total load)	Imbalance (%)
0.0	49.8	49.9	49.8	49.8	0.1
0.5	70.4	75.1	71.2	72.2	4.0
1.0	78.5	100.4	92.6	90.5	13.2
1.5	111.8	125.8	114.0	117.2	7.4
2.0	132.6	151.3	135.5	139.8	8.2

In tables 1-2 imbalance is calculated from

$$Imbalance = \frac{\max\{P_i - average(P_i)\}}{average(P_i)} \quad i = a, b \text{ and } c \tag{2}$$

From tables 1-2 it may be noted that as the DRs voltage magnitude or angle are increased, the imbalance in the feeder phase power flow increases. This imbalance is due to the unbalanced impedance and admittance of the distribution feeder, where the unbalanced phase impedance matrix is presented in Eqn. 1 above. It may be noted that if the DR's voltage magnitude is increased by 4% above that of the substation's voltage magnitude, then the DR supplies all of the feeder load on phase A, 95% of the feeder load on phase B, and 83% of the feeder load on phase C. A point of interest is that the operation of a balanced source (with balanced loads) is causing significant unbalanced power flows in the feeder due to the unbalanced impedance.

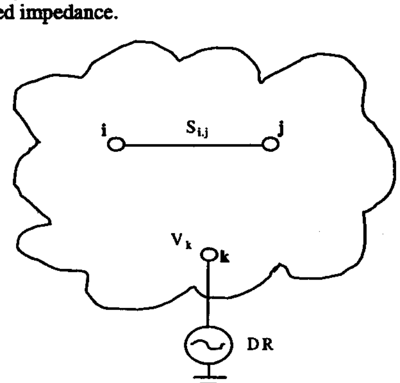

Figure 2 DR Terminal Voltage at Node k and Complex Power Flow in Line ij for General Power System

An analytic expression will now be derived that relates a change in power flow in a line section to a change in a DRs terminal voltage. This expression will show the relation between unbalanced impedances and unbalanced power flows, and may be used to verify the power flow results observed.

The objective of our analysis is to find the relation between a change in voltage at the DR terminals and a change in a line flow in the system. For example, consider Figure 2 where it is desired to determine the change in the complex power flows in line ij for all three phases when there is a change in the terminal voltage of the DR at node k.

Letting $\bar{S}$ represent a complex power flow, $\bar{V}$ a phasor voltage, $\bar{I}$ a phasor current, and asterisk(*) the congugate of a complex number, consider the following per phase equations in relation to the ij power line shown in Figure 2.

$$\begin{bmatrix} \bar{S}^*_{ij-a} \\ \bar{S}^*_{ij-b} \\ \bar{S}^*_{ij-c} \end{bmatrix} = \begin{bmatrix} \bar{V}^*_{i-a} & 0 & 0 \\ 0 & \bar{V}^*_{i-b} & 0 \\ 0 & 0 & \bar{V}^*_{i-c} \end{bmatrix} \cdot \begin{bmatrix} \bar{I}_{ij-a} \\ \bar{I}_{ij-b} \\ \bar{I}_{ij-c} \end{bmatrix} \tag{3}$$

Rewriting in vector-matrix form gives

$$\bar{\mathbf{S}}^*_{ij} = \left[\bar{\mathbf{V}}^*_i\right] \cdot \bar{\mathbf{I}}_{ij} \tag{4}$$

Assuming that the voltage at node i remains constant for a small change in the terminal voltage of the DR gives

$$\Delta\bar{\mathbf{S}}^*_{ij} = \left[\bar{\mathbf{V}}^*_i\right] \cdot \Delta\bar{\mathbf{I}}_{ij} = \left[\bar{\mathbf{V}}^*_i\right] \cdot \left[z_{ij}\right]^{-1} \cdot \Delta(\bar{\mathbf{V}}_i - \bar{\mathbf{V}}_j) \tag{5}$$

where $[z_{ij}]$ is the 3x3 impedance matrix of line ij, such as Eqn. 1.

Now let $[Z_{ik}]$, $[Z_{jk}]$, and $[Z_{kk}]$ be system bus impedance matrices, such that

$$\bar{\mathbf{V}}_i - \bar{\mathbf{V}}_j = \left[Z_{ik}\right] \cdot \bar{\mathbf{I}}_k - \left[Z_{jk}\right] \cdot \bar{\mathbf{I}}_k = \left\{\left[Z_{ik}\right] - \left[Z_{jk}\right]\right\}\left[Z_{kk}\right]^{-1} \cdot \bar{\mathbf{V}}_k \tag{6}$$

Combining Eqns. 5 and 6, we will have

$$\Delta\bar{\mathbf{S}}^*_{ij} = \left\{\left[\bar{V}^*_i\right]\left[z_{ij}\right]^{-1}\left\{\left[Z_{ik}\right] - \left[Z_{jk}\right]\right\}\left[Z_{kk}\right]^{-1}\right\} \cdot \Delta\bar{\mathbf{V}}_k \tag{7}$$

Equation 7 may be used to verify the power flow results shown in tables 1 and 2.

Modeling the DR as a balanced voltage source does not take into account the interaction between the DR and the system. Realistically, in order to have an effect on its terminal voltage, the DR will have to interact with the power system. This will be considered in the next section.

III. SYNCHRONOUS GENERATOR MODEL FOR UNBALANCED CIRCUIT CALCULATIONS

The model developed here for the synchronous generator will no longer assume that the terminal voltage magnitudes and angles can be controlled to any value. Here the synchronous generator can affect the terminal voltage values only through an interaction with the distribution power system itself. The solution of the distribution system will determine the machine's terminal voltages, and futhermore the terminal voltages may be unbalanced. The currents injected by the machine will be a function of the terminal voltages, and hence the injected currents may also be unbalanced. If the imbalance in the injected currents becomes too large, the machine may be shut down by its protection system. Only small imbalances in injected currents are considered here. Note that the solution of the distribution system will set the machine's terminal voltages, but the machine in turn can affect these terminal voltages by varying the injected currents.

Both single-phase and 3-phase synchronous generators may serve as distributed resources. The model developed here will be for a 3-phase synchronous generator.

Equations 8-10 shown below, based on Faraday's Law, will be the starting point for the model development [5,6].

$$v_a = -\frac{d}{dt}\lambda_a \quad (8)$$

$$v_b = -\frac{d}{dt}\lambda_b \quad (9)$$

$$v_c = -\frac{d}{dt}\lambda_c \quad (10)$$

where v_k = voltage of phase k
λ_k = flux linkages for phase k
k = a, b, c.

Assuming a round rotor synchronous generator, the flux linkages may be calculated from

$$\lambda_a = L_S \cdot \left(2i_a - i_b - i_c\right) + L_{SR} \cdot I_R \cdot \cos\left(\omega_s t\right) \quad (11)$$

$$\lambda_b = L_S \cdot \left(-i_a + 2i_b - i_c\right) + L_{SR} \cdot I_R \cdot \cos\left(\omega_s t - 120\right) \quad (12)$$

$$\lambda_c = L_S \cdot \left(-i_a - i_b + 2i_c\right) + L_{SR} \cdot I_R \cdot \cos\left(\omega_s t + 120\right) \quad (13)$$

where i_k = stator current for phase k
I_R = rotor current,
L_S = self-inductance of stator
L_{SR} = mutual-inductance between stator and rotor.

Assuming that the phase currents may have different magnitudes and phase angles we may write

$$i_a = I_A \cos\left(\omega_s t + \varphi_A\right) \quad (14)$$

$$i_b = I_B \cos\left(\omega_s t + \varphi_B\right) \quad (15)$$

$$i_c = I_C \cos\left(\omega_s t + \varphi_C\right) \quad (16)$$

Substituting Eqns. 14-16 into Eqn. 11 results in

$$\lambda_a = 2L_S I_A \cos\left(\omega_s t + \varphi_A\right) - L_S I_B \cos\left(\omega_s t + \varphi_B\right) - L_S I_C \cos\left(\omega_s t + \varphi_C\right) + L_{SR} I_R \cos\left(\omega_s t\right) \quad (17)$$

Substituting Eqn. 17 into Eqn. 8 gives

$$v_a = 2\omega_s L_S I_A \sin\left(\omega_s t + \varphi_A\right) - \omega_s L_S I_B \sin\left(\omega_s t + \varphi_B\right) - \omega_s L_S I_C \sin\left(\omega_s t + \varphi_C\right) + \omega_s L_{SR} I_R \sin\left(\omega_s t\right) \quad (18)$$

Applying a phasor transformation to Eqn. 18 gives

$$V_A e^{j\theta_A} = \frac{2\omega_s L_S I_A}{\sqrt{2}} e^{j\varphi_A} - \frac{\omega_s L_S I_B}{\sqrt{2}} e^{j\varphi_B} - \frac{\omega_s L_S I_C}{\sqrt{2}} e^{j\varphi_C} + \frac{\omega_s L_{SR} I_R}{\sqrt{2}} e^{j0} \quad (19)$$

where V_A = voltage magnitude of phase A
θ_A = voltage angle of phase A
I_k = current magnitude for phase k
φ_k = phase current angle of phase k.

Defining

$$X_s = \frac{\omega_s L_S}{\sqrt{2}} \quad (20)$$

$$V_f = \frac{\omega_s L_{SR} I_R}{\sqrt{2}} \quad (21)$$

Substituting Eqns. 20 - 21 into Eqn. 19 gives

$$V_A e^{j\theta_A} = 2X_s I_A \cdot e^{j\varphi_A} - X_s I_B e^{j\varphi_B} - X_s I_C e^{j\varphi_C} + V_f e^{j0} \quad (22)$$

Performing a similar derivation for phases B and C as was done for phase A gives

$$V_B e^{j\theta_B} = -X_s I_A e^{j\varphi_A} + 2X_s I_B e^{j\varphi_B} - X_s I_C e^{j\varphi_C} + V_f e^{-j120} \quad (23)$$

$$V_C e^{j\theta_C} = -X_s I_A e^{j\varphi_A} - X_s I_B e^{j\varphi_B} + 2X_s I_C e^{j\varphi_C} + V_f e^{j120} \quad (24)$$

Equations 22 - 24 represent a phasor model of a 3-phase synchronous generator, where the terminal voltage and current magnitudes are not necessarily equal.

Equations 22 – 24 are the basis for the DR simulation to be performed in the next section. The terminal voltages of the DR will be determined from a multi-phase power flow solution. The phase currents injected by the DR will be a function of the terminal voltages and may be calculated from Eqns 22-24.

IV. UNBALANCED CIRCUIT SIMULATION

Figure 3 shows the circuit to be used for the simulation studies. The circuit consists of a substation, five cable sections, and a distributed resource that is connected to the circuit through the switch labeled "SW1." Thus, the DR injects power into the circuit at Node 2.

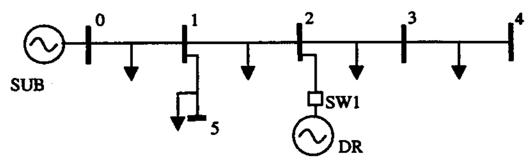

Figure 3 Distributed Resource Modeled as Voltage-Dependent Current Source

Each cable section shown in Figure 3 consists of a 3-phase cable, and the length of all cable sections is the same. The impedance matrix for each cable section is given by

$$\left[z_{ij}\right] = \begin{bmatrix} 0.20+j0.50 & 0.10+j0.10 & 0.10+j0.10 \\ 0.10+j0.10 & 0.20+j0.50 & 0.10+j0.10 \\ 0.10+j0.10 & 0.10+j0.10 & 0.20+j0.50 \end{bmatrix} \quad (25)$$

The loads for each cable section shown in Figure 3 are given in Table 3. Note that there is a significant imbalance in the loading on phases A, B, and C. This loading imbalance will also cause a voltage imbalance to exist among the phases at each node. Thus, the DR located at node 2 will be working against a set of unbalanced phase voltages.

Table 3 Loads for Circuit Shown in Figure 3

	Phase A (kW + j kVAR)	Phase B (kW + j kVAR)	Phase C (kW + j kVAR)
Cable (0,1)	1250+j1250	625+j625	312.5+j312.5
Cable (1,2)	0+j0	0+j0	0+j0
Cable (2,3)	1250+j1250	625+j625	312.5+j312.5
Cable (3,4)	1250+j1250	625+j625	312.5+j312.5
Cable (1,5)	1250+j1250	625+j625	312.5+j312.5

Two simulation studies are to be considered. In the first study the switch SW1, shown in Figure 3, is open and thus the DR is switched out of the circuit. In the second study the switch SW1 is closed and the DR supplies power to the circuit. The size of the DR was set at 4.5 MW, and thus represents approximately 50% of the existing circuit power requirements.

The results of the simulation runs are presented in Table 4. The first column of the table presents the node number, corresponding to the nodes shown in Figure 3, for which the row results apply. The next three columns present the customer level voltage (in volts) for each of the phases A, B, and C, where the DR is not switched into the circuit. Following that, the next three columns present customer level voltage for each of the phases A, B, and C, where the DR is switched into the circuit. The final three columns show the change in the voltage magnitude for each phase that occurs as a result of switching the DR into the circuit.

Table 4 Customer Level PhaseVoltages for Circuit Shown in Figure 3 With and Without DR Operating

Node	SW1 Open			SW1 Closed			Change of |V|		
	A	B	C	A	B	C	A	B	C
0	127.02	127.02	127.02	127.02	127.02	127.02	0.00	0.00	0.00
1	120.13	124.46	126.77	121.92	126.15	128.33	1.79	1.69	1.56
2	116.70	123.19	126.66	120.25	126.55	129.79	3.55	3.36	3.13
3	113.29	121.94	126.54	116.84	125.30	129.67	3.55	3.36	3.13
4	111.58	121.32	126.48	115.13	124.67	129.61	3.55	3.35	3.15

Assume that the nominal customer level voltage is 120 volts, and that the voltage limits at loads are set at 110 volts on the low side and 128 volts on the high side. In light of these voltage constraints, consider the simulation results for the case where SW1 is open. In this case no voltage constraints are violated.

After the DR is switched into the circuit and supplies power, the phase C voltage at nodes 1-4 goes out of limits. Hence, switching the DR into the circuit caused voltage limits to be violated, where no violations existed previously.

One way the voltage violations could be caused in practice is via a voltage magnitude control that has a feedback measurement existing on only a single-phase. This type of control, where only a single-phase measurement is made, is common in distribution systems. For instance, in the simulation study considered here, assume that it is desired that the DR control the phase A voltage to 120 volts. Such a control would result in the phase C voltage at nodes 1-4 going out of limits.

It should be noted that the DR may be used to raise the voltage magnitudes along the line. This is due to the changes in the line flows upstream (i.e., toward the substation) from the DR. Note that downstream of the DR the voltages are pulled up uniformly. An analytical formula for evaluating such a voltage control is derived in the next section. The formula may be used to check the simulation results presented here.

V. DERIVATION OF ANALYTICAL FORMULAS

Consider Figure 4, where voltages and power flows are shown for a power line with a DR installed at node 2.

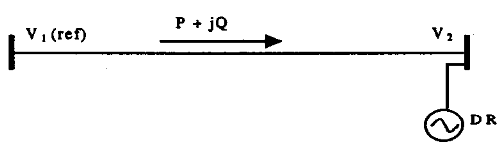

Figure 4 Voltages and Power Flows for Line with DR at Ending Node

For the present, assume that Figure 4 represents a single-phase line. Given node 1 as reference, i.e., $\bar{V}_1 = Ve^{j0}$, the node 2 voltage of Figure 4 may be expressed as

$$\bar{V}_2 = V - \frac{1}{V}\left[P\cdot R + Q\cdot X\right] + j\frac{1}{V}\left[Q\cdot R - P\cdot X\right] = (V + \Delta V) + j(\delta V) \qquad (26)$$

where $R + jX$ = line impedance.

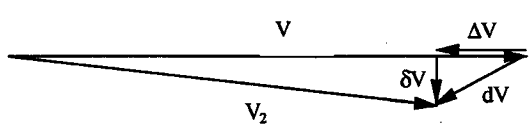

Figure 5 Phase Voltage Relationships for Power Line Shown in Figure 4

Figure 5 shows the phasor relationships between the voltages of Eqn. 26. Note that the change of V_2 is primarily due to ΔV and not δV. Thus, we may write the following

From Eqn. 26, V_2, the magnitude of voltage at node 2, is given by

$$V_2 \approx V + \Delta V = V - \frac{1}{V}\left[P\cdot R + Q\cdot X\right] = \mathrm{Re}(\bar{V}_2) \qquad (27)$$

From Eqn. 27, it may be noted that the magnitude of the ending node voltage primarily depends upon ΔV, the real part of the voltage drop. The angle of the ending node voltage primarily depends upon δV, the imaginary part of the voltage drop (see Figure 5).

Assuming $\bar{V}_1$ stays constant and that there is a decrease in the power flows in the line, ΔP and ΔQ, then the voltage magnitude at node 2 will be given by

$$V_2' = V - \frac{1}{V}\left[(P-\Delta P)\cdot R + (Q-\Delta Q)\cdot X\right] \qquad (28)$$

Subtracting Eqn. 27 from Eqn. 28

$$dV_2 = V_2' - V_2 = \mathrm{Re}\left(\bar{V}_2'\right) - \mathrm{Re}\left(\bar{V}_2\right) = \mathrm{Re}\left(\bar{V}_2' - \bar{V}_2\right) = \frac{1}{V}\left[\Delta P\cdot R + \Delta Q\cdot X\right] \qquad (29)$$

If there is a power injection at node 2, for instance from a DR, then the P and Q line flows will decrease in proportion to the power injection. Hence, $(V'_2 - V_2)$ will be positive and the voltage magnitude at node 2 will increase.

Now assume that the power line in Figure 4 is a 3-phase line with symmetric mutual coupling, where the impedance matrix for the power line may be written as

$$Z = \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} \qquad (30)$$

Also assume that $\bar{V}_1$ for phases A, B and C has the same magnitude and has angles of 0, -120 and 120 degrees, respectively. We will also assume that $\bar{V}_1$ stays constant, such as the voltage at the substation. Let $\bar{V}_1$ be given as

$$\bar{V}_{1\_abc} = \begin{bmatrix} \bar{V}_{1\_a} \\ \bar{V}_{1\_b} \\ \bar{V}_{1\_c} \end{bmatrix} = \begin{bmatrix} V \\ V\cdot e^{-j120} \\ V\cdot e^{j120} \end{bmatrix} \qquad (31)$$

Then, the voltage at node 2 is given as

$$\begin{bmatrix} \bar{V}_{2\_a} \\ \bar{V}_{2\_b} \\ \bar{V}_{2\_c} \end{bmatrix} = \begin{bmatrix} V \\ V\cdot e^{-j120} \\ V\cdot e^{j120} \end{bmatrix} - \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} \cdot \begin{bmatrix} \frac{P_a - jQ_a}{V} \\ \frac{P_b - jQ_b}{V\cdot e^{j120}} \\ \frac{P_c - jQ_c}{V\cdot e^{-j120}} \end{bmatrix} \qquad (32)$$

We will first analyze the voltage at node 2 for phase A. The voltages for phases B and C are similar except for phase angle shifts of –120 and 120 degrees, respectively. From Eqn. 31, the voltage at node 2 for phase A is given by

$$\bar{V}_{2\_a} = V - \frac{1}{V}\left[Z_s\left(P_a - jQ_a\right) + Z_m\left(P_b - jQ_b\right)e^{-j120} + Z_m\left(P_c - jQ_c\right)e^{j120}\right] \qquad (33)$$

Now assume the DR injects $\Delta P + j\Delta Q$ for each phase. This will decrease the line flow by $\Delta P + j\Delta Q$ for each phase, where we neglect the effects of power loss. This can be expressed as

$$P_k' + jQ_k' = \left(P_k - \Delta P\right) + j\left(Q_k - \Delta Q\right) \qquad (34)$$

where $P'_k + jQ'_k$ = line flow of phase k

Therefore, the voltage change at node 2 for phase A is given by

$$\bar{V}'_{2\_a} - \bar{V}_{2\_a}$$
$$= \frac{1}{V}\left[Z_s \cdot (\Delta P - j\Delta Q) + Z_m \cdot (\Delta P - j\Delta Q) \cdot e^{-j120} + Z_m \cdot (\Delta P - j\Delta Q) \cdot e^{j120}\right]$$
$$= \frac{1}{V}\left[\left(Z_s + Z_m e^{-j120} + Z_m e^{j120}\right) \cdot (\Delta P - j\Delta Q)\right]$$
$$= \frac{1}{V}\left[(Z_s - Z_m) \cdot (\Delta P - j\Delta Q)\right]$$
$$= \frac{1}{V}\left[(R_s - R_m) \cdot \Delta P + (X_s - X_m) \cdot \Delta Q\right] + j\frac{1}{V}\left[(X_s - X_m) \cdot \Delta P - (R_s - R_m) \cdot \Delta Q\right] \quad (35)$$

The change of voltage magnitude at node 2 of phase A is the real part of ($\bar{V}'_{2\_a} - \bar{V}_{2\_a}$). Thus, the change of voltage magnitude is given by

$$dV_{2\_a} = V'_{2\_a} - V_{2\_a} = \mathrm{Re}\left(\bar{V}'_{2\_a}\right) - \mathrm{Re}\left(\bar{V}'_{2\_a}\right)$$
$$= \mathrm{Re}\left(\bar{V}'_{2\_a} - \bar{V}_{2\_a}\right) = \frac{1}{V}\left[\Delta P \cdot (R_s - R_m) + \Delta Q \cdot (X_s - X_m)\right] \quad (36)$$

Similar to the derivation from Eqn. 31 - 36, the change of voltage magnitude for phases B and C are given by

$$dV_{2\_b} = dV_{2\_c} = \frac{1}{V}\left[\Delta P \cdot (R_s - R_m) + \Delta Q \cdot (X_s - X_m)\right] \quad (37)$$

Equations 36 and 37 give the same results for each phase. This is quite reasonable because everything for phases B and C are identical to phase A except that there is a –120 or 120 degree phase angle shift. In the above analysis, we assumed the voltage at node 1 shown in Figure 4 stays constant.

Now consider Figure 6, with a DR connected at node 3. The voltages at nodes 1 and 2, upstream of node 3, will change due to a change in the DRs power injection. Then, the overall voltage change at node 3 is the accumulation of voltage changes at nodes 1, 2, and 3.

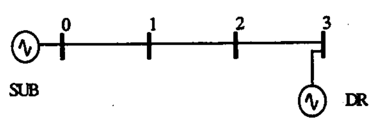

Figure 6 Accumulation of Voltage Change in DR's upstream lines

For a general node n where a DR injects ΔP+jΔQ, assume that the power flow in all upstream lines of node n will be decreased by ΔP+jΔQ (where changes in power loss are neglected). Then, the expected change in voltage magnitude at node n is given by

$$dV_{n\_a} = dV_{n\_b} = dV_{n\_c}$$
$$= \frac{\Delta P}{V} \cdot \sum_{i \in U}\left(R_{s,i} - R_{m,i}\right) + \frac{\Delta Q}{V} \cdot \sum_{i \in U}\left(X_{s,i} - X_{m,i}\right) \quad (38)$$

where V is the rated voltage level or voltage base
U is the set of all lines upstream of node n.

If we consider all lines from node 0 (the substation) to node n (the DR node) as a "long" power line, Eqn. 38 will be exactly Eqn.37.

By substituting the appropriate parameter and variable values into Eqn. 38, the simulation results obtained earlier in this chapter may be verified. Also, Eqn. 38 may be used by a system planner to place a DR in a distribution circuit for voltage magnitude control.

VI. CONCLUSIONS

This paper has addressed the development of DR models for use in unbalanced power flow analysis of electrical distribution circuits.

The results from section II show that a DR, modeled as a balanced voltage source, in a distribution circuit with perfect load balance can result in unbalanced power flows due to the unbalanced line impedances. The voltage source approximation of the DR is only valid for small variations, but illustrates power flow interactions that would occur between the substation, viewed as a voltage source, and a DR whose terminal voltages deviate from that of the substation. An analytical formula for a 3-phase system was developed that relates changes in real and reactive power flows to changes in DR source voltage magnitudes and angles.

A three-phase model for a DR of type synchronous generator was developed. The model was then used in a simulation study. The simulation showed that in a distribution circuit operating in an unbalanced but legal state, a DR being switched on can cause circuit constraints to be violated. Hence, the control of DRs under unbalanced circuit conditions could be important. Analytical formulas were developed which may be used to confirm the simulation results and which illustrate the voltage magnitude control capabilities of DRs.

In summary, conclusions that can be drawn from this work are

- DRs with balanced terminal voltages can cause significant imbalances in distributed circuits
- When a DR is switched into a circuit in which there are no constraint violations existing prior to the switching operation, the DR can cause circuit voltage constraints to be violated.

VII. ACKNOWLEDGEMENTS

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IX. BIOGRAPHIES

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